TECHNICAL NOTE

# Laminar film condensation inside a horizontal elliptical tube with variable wall temperature

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An analytical study is performed for a condensation heat transfer with the quiescent or very slow vapor inside a horizontal elliptical tube in which the wall temperature $T_w$ may vary along the circumferential coordinates. Analytical solutions for local condensate film thickness, local and mean heat-transfer coefficients have been expressed in terms of ellipticity $e$ of the cross section, each of wall temperature variation functions and the reverse Bond numbers. The results show that both surface tension forces and wall temperature variation functions affect the local heat-transfer coefficient and hydrodynamics characteristics inside an elliptical tube; however, the surface tension effect on the mean heat-transfer coefficient is nearly insignificant. The variation of $\overline{Nu}$ with the amplitude of wall temperature variation functions depends on the position forming the liquid level.

**Keywords:** horizontal elliptical tube; non-isothermal wall; laminar film condensation

## Introduction

Condensation heat-transfer inside horizontal or inclined tubes with low vapor shear force was first studied by Chaddock (1957) and Chato (1960). In recent years, this field has been applied in heat-pipe, thermosyphon, and waste heat recovery (Moalem and Sideman 1976; Wang and Ma 1991). Besides, from the mathematical point of view, a circular tube is one kind of elliptical tube with zero ellipticity; a flat plate is another kind of elliptical tube with ellipticity equal to 1. Hence, in our study (1993), the condensing heat transfer rate on a vertical plate is better than that on a circular tube. Consequently, an elliptical tube with its major axis oriented in gravity direction is expected to possess a better heat transfer than a circular tube.

Although the analytical solution regarding the mean Nusselt number of film condensation inside the circular tube can be found in Collier's book (1981), the similar solution for a more general elliptical geometry is not yet known. As for the film condensation outside the circular tube with variable wall temperature (a cosine distribution), Memory and Rose (1991) found that the local condensate film thickness and heat flux depend markedly on the amplitude of the surface temperature variation; however, the mean heat-transfer coefficient is virtually unaffected by surface temperature non-uniformity.

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The major aim of this note is intended to help the easy use of the Nusselt-Rohsenow-type condensation analysis inside general elliptical tubes with nonisothermal wall including further account of the effect of surface tension by developing analytical, explicit, and straightforward integrating solutions.

## Analysis

Consider a horizontal elliptical tube with its major axis "$2a$" oriented in the direction of gravity, filled with a quiescent or very slow-flowing pure vapor, which is at its saturation temperature $T_{sat}$. The wall temperature $T_w$ is nonuniform and below the saturation temperature. Thus, condensation occurs on the wall, and a continuous film of the liquid runs downward over the tube under the actions of the component of gravity and the surface tension forces. The condensate will eventually collect as a stratified layer of liquid in the lower part of the tube and then flows off in a kind of open-channel flow, especially for the low condensation rates or slow flowing vapor within short tubes, as illustrated in Figure 1.

By assuming that the condensate is a laminar, steady-state flow with constant fluid properties, the Nusselt's local mass flow rate is expressible in terms of the local film thickness $\delta(x)$ without reference to the history of the film up to this location as

$$\dot{m} = \rho(\rho - \rho_v)\frac{q\delta^3}{3\mu}[\sin\phi - Bo(\phi)] \tag{1}$$

where $\phi = \phi(x)$ is the angle between the normal to gravity and the tangent to the tube wall at the position $(r, \theta)$. Here, $\theta$ is the

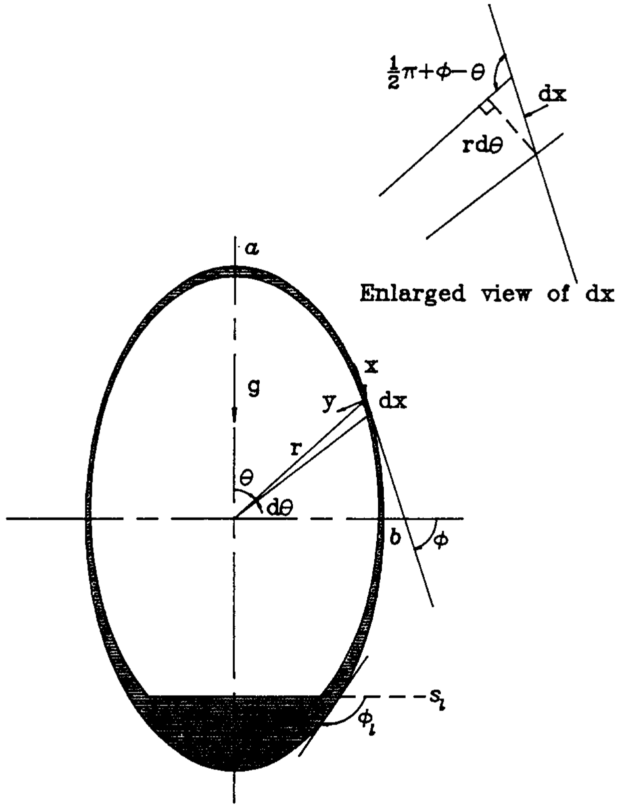


*Figure 1* Schematic and coordinate system

angle measured from the tube upper generatrix; $r$ is the radial distance from the centroid of the ellipse and can be expressed as

$$r = a[(1 - e^2)/(1 - e^2 \cos^2 \theta)]^{0.5} \tag{2}$$

where $e = \sqrt{a^2 - b^2}/a$ is an ellipticity of the ellipse. Additionally, the pressure gradient resulting from surface tension can be derived as

$$Bo(\phi) = \frac{3e^2}{2Bo}\left(\frac{1 - e^2 \sin^2 \phi}{1 - e^2}\right)^2 \sin(2\phi) \tag{3}$$

and $Bo = (\rho - \rho_v)\, ga^2/\sigma$, the Bond number.

The heat flux at the liquid-vapor interface is related to the rate of condensation by

$$h_{fg}\frac{d\dot{m}}{dx} = k\left.\frac{dT}{dy}\right|_{y=0} = k\frac{\Delta T}{\delta} \tag{4}$$

where $\dot{m}$ is the rate of the condensate mass flow over an elliptical perimeter per unit tube length. To derive the local film thickness $\delta$ at the circumferential arc length $x$ (or angle $\theta$) in terms of $\phi$, one can substitute Equation 5 into Equation 4 and obtain

$$\frac{\rho(\rho - \rho_v)g}{3\mu}h_{fg}\frac{d}{dx}\{\delta^3[\sin \phi - Bo(\phi)]\} = \frac{k}{\delta}\Delta T \tag{5}$$

It is more convenient at this point to express $dx$ in polar coordinates. With reference to Figure 1, the differential elliptical arc length may be written as

$$dx = \frac{r\, d\theta}{\cos(\phi - \theta)} \tag{6}$$

By using the properties of an ellipse it may be shown that the tangent at any point is given as

$$\tan \phi = \tan \theta/(1 - e^2) \tag{7}$$

To compare the condensate film-flow hydrodynamics and heat-transfer characteristics with circular tubes, based on the same condensing surface area or same perimeter per unit length of tube, one may introduce an equivalent diameter $D_e$ as

$$D_e = 2\frac{a}{\pi}\int_0^{\pi}[(1 - e^2)/\sqrt{(1 - e^2 \sin^2 \phi)^3}]\, d\phi \tag{8}$$

and dimensionless streamwise length $s$ as

$$s = \int_0^{\phi}(1 - e^2 \sin^2 \phi)^{-3/2}\, d\phi \bigg/ \int_0^{\pi}(1 - e^2 \sin^2 \phi)^{-3/2}\, d\phi \tag{9}$$

Once the wall temperature distribution is specified or fitted by the experimental data, one may calculate the mean wall temperature as

$$\overline{T_w} = \frac{2}{\pi}\frac{a}{D_e}\int_0^{\pi} T_w(\phi)[(1 - e^2)/\sqrt{(1 - e^2(\sin^2 \phi)^3}]\, d\phi \tag{10}$$

and express the temperature difference across the film as

$$T_{sat} - T_w = (T_{sat} - \overline{T_w})F_t(\phi) = \overline{\Delta T}\, F_t(\phi) \tag{11}$$

$F_t(\phi)$, the wall temperature variation function, is the dimensionless temperature profile in the circumferential direction.

Substituting Equations 6 and 11 into Equation 5, and introducing transformation of variable from $x$ to $\phi$, one obtains the local film thickness at $\phi$ as follows:

$$\delta^* = \delta\left[\frac{D_e k\mu\overline{\Delta T}/g}{h_{fg}\rho(\rho - \rho_v)}\right]^{-1/4}$$
$$= I_f(\phi)\left\{\frac{1}{\pi}\int_0^{\pi}[(1 - e^2)/\sqrt{(1 - e^2 \sin^2 \phi)^3}]\, d\phi\right\}^{-1/4} \tag{12}$$

## Notation

| | |
|---|---|
| $A$ | Amplitude of the wall temperature variation functions |
| $a, b$ | Semimajor, semiminor axis of ellipse |
| $C_p$ | Specific heat of condensate at constant pressure |
| $g$ | Acceleration because of gravity |
| $h, \bar{h}$ | Local, mean condensing heat-transfer coefficient |
| $h_{fg}$ | Latent heat of condensation |
| $k$ | Thermal conductivity of condensate |
| $Nu, \overline{Nu}$ | Local, mean Nusselt number |
| $Pr$ | Prandtl number |
| $x, y$ | Coordinate measuring length along circumference from top of tube, normal to $x$ |
| $\delta, \delta^*$ | Local, dimensionless thickness of condensate film |
| $\mu$ | Absolute viscosity of condensate |
| $\rho, \rho_v$ | Density of condensate, vapor |
| $\sigma$ | Surface tension coefficient in the film |
| $\phi$ | Angle between the tangent to tube surface and the normal to direction of gravity |
| $\phi_1$ | Angle forming the liquid level |

where

$$I_f = [\sin\phi + Bo(\phi)]^{-1/3} \times \left\{2(1-e^2)\int_0^{\phi} F_t(\phi)[\sin\phi + Bo(\phi)]^{1/3} \times (1-e^2\sin^2\phi)^{-3/2}\,d\phi\right\}^{1/4}$$

In terms of condensing heat transfer, interpreting the result of the model is straightforward by employing the usual idea of the local heat-transfer coefficient as follows:

$$Nu = \frac{hD_e}{k} = [Ra/Ja]^{1/4}/\delta^* \tag{13}$$

where, Rayleigh number, $Ra = \rho(\rho - \rho_v)gPrD_e^3/\mu^2$ and Jakob number, $Ja = C_p\overline{\Delta T}/h_{fg}$.

From Equations 1, 4, 6, 7, and 11, one may derive the condensate production above the liquid level (at $\phi_1$, the angle forming the liquid level, see Figure 1) from one side as

$$\dot{m} = \frac{1}{3}\left[\frac{64k^3a^3\overline{\Delta T}^3\rho(\rho-\rho_v)g}{\mu(h_{fg})^3}\right]^{1/4} \times \left\{(1-e^2)\int_0^{\phi_1}\frac{F_t(\phi)[Bo(\phi)+\sin\phi]^{1/3}}{(1-e^2\sin^2\phi)^{3/2}}\,d\phi\right\}^{3/4} \tag{14}$$

Noting that the preceding relation gives only half of the condensate mass flow from the tube, one finds that an energy balance within the condensate film over an entire elliptical perimeter per unit tube length yields

$$2\dot{m}\,h_{fg} = \bar{h}(s_1\pi D_e)\overline{\Delta T} \tag{15}$$

Inserting Equation 14 into Equation 15, one may obtain the dimensionless mean heat-transfer coefficient as

$$\overline{Nu}_{\phi_1} = \frac{\bar{h}D_e}{k} = \left(\frac{128}{81\pi}\right)^{1/4}[Ra/Ja]^{1/4}S_f(\phi_1)/s_1 \tag{16}$$

where

$$S_f(\phi_1) = \left\{\int_0^{\phi_1}\frac{F_t(\phi)[\sin\phi + Bo(\phi)]^{1/3}}{(1-e^2\sin^2\phi)^{3/2}}\,d\phi \div \int_0^{\pi}(1-e^2\sin^2\phi)^{-3/2}\,d\phi\right\}^{3/4}$$

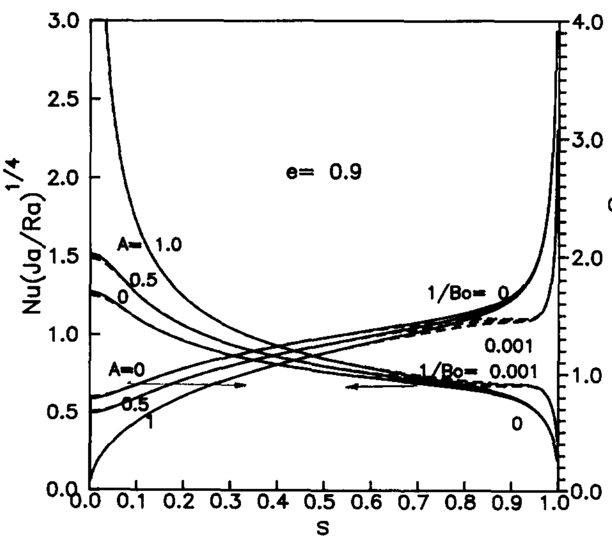


*Figure 2* Dimensionless local film thickness and heat-transfer coefficient around periphery of ellipse

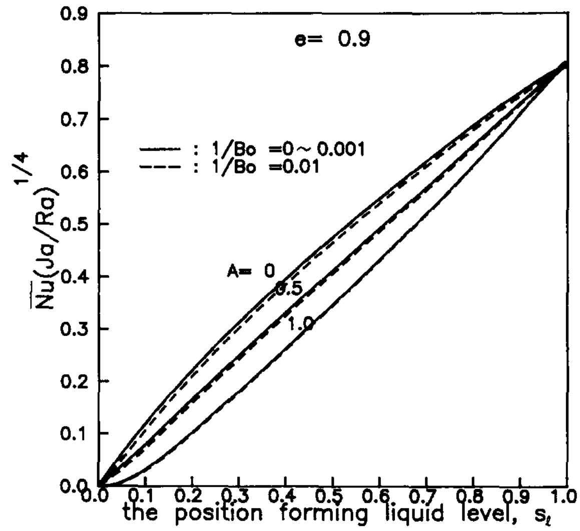


*Figure 3* Dependence of overall mean heat transfer coefficient on the position forming liquid level

Therefore, the overall mean Nusselt number may be expressed as

$$\overline{Nu} = s_1\overline{Nu}_{\phi_1} + (1-s_1)\overline{Nu}_{liq} \tag{17}$$

Note that, according to Chato's (1960) study in the case of circular tube, $\overline{Nu}_{liq} \approx 0$, which is due to a much larger film thickness below the liquid level. Similarly, for the present cases of elliptical tubes, one may obtain the overall mean Nusselt number by the same manner.

## Results and discussion

According to Lee et al.'s (1984) experimental results for laminar film condensation on a horizontal circular tube ($e = 0$), the wall temperature variation distribution for $e \neq 0$ can be similarly assumed or fitted by the following nonisothermality function:

$$F_t(\phi) = 1 - A\cos(\phi) \tag{18}$$

It is to be noted that $0 \le A \le 1$; the amplitude $A$ depends largely on the ratio of the outside-to-inside heat-transfer coefficients. When $A = 0$, the wall temperature is uniform.

Equations 12 and 13 have been evaluated numerically for different values of ellipticity and the nonisothermality function at particular angular position $\phi$, or its corresponding dimensionless circumferential position. It is seen that the local film thickness decreases as $A$ increases, whereas the local Nusselt number increases as $A$ increases in Figure 2. Additionally, in the upper half of the tube, the pressure gradient term is negative ($-\partial P/\partial x < 0$), the effect of surface tension forces tend to retard the drainage of liquid film, and thus keeps film thicker. However, in the lower half of the tube, the pressure gradient is positive because of the increasing surface curvature, so the condensate film will become thinner.

In Figure 3, one may see that the overall mean heat-transfer coefficient decreases remarkably with increasing $A$ at both ends: $s_1 = 0$ and $s_1 = 1.0$. Besides, the overall mean heat-transfer coefficient become less dependent on the wall temperature variation of cosine distribution for any elliptical tube, as $s_1$ more approaches 1 (i.e., bottom). In Figure 4, if one neglects the surface tension effect, one will overestimate the mean heat-transfer coefficient slightly.

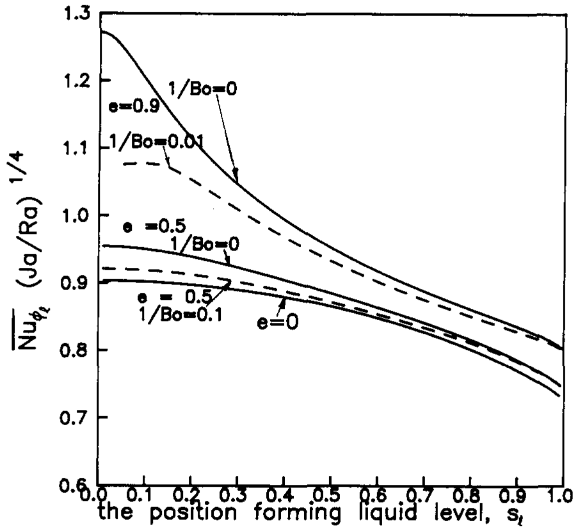


*Figure 4* Dependence of mean heat-transfer coefficient on the position forming liquid level

In engineering applications, the angle forming the liquid level may be specified or measured from the practical system by the operating conditions, such as the total length of the tube and vapor flow velocities. For example: $\phi_1 = 2\pi/3$, $s_1 = 0.8521$, $e = 0.9$ and $F_t(\phi) = 1.0$, one has

$$\overline{Nu} = 0.7228\,(Ra/Ja)^{1/4} \tag{19}$$

which is larger than that of a circular tube for the same operating conditions, $\phi_1 = 2\pi/3$. But its use in industry may still not be justified because of the possible increased cost in manufacturing.

The alternative and perhaps more common case is when there exists a pressure gradient and the condensate at the tube outlet fills the elliptical tube cross section. The angle forming the liquid level $\phi_1$ decreases with the tube length. For an elliptical tube with the particular ellipticity $e$ in Figure 3, one may see that the overall mean heat-transfer coefficient decreases with the tube length because the angle forming the liquid level $\phi_1$ decreases with the tube length.

The present model, which assumes that the vapor is stagnant or very slow moving, can apply to the thermosyphon and heat pipe, and near the position where the tube becomes completely filled with liquid. Slug, plug, and wavy flow patterns, although still basically stratified, correspond to situations in which the vapor shear forces are significant, because axial vapor velocities are often high, especially near to inlet. Analyses of these flow patterns should therefore include the further effect of vapor shear.

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